# The Cannon Problem

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# ME498/599 Ballistic Problem with Air Resistance

- Sensitivity analysis
- Uncertainty quantification
- Uncertainty propagation

# Math Model

- The equations of motion (Newton's Second Law: $ma = \sum F$) are:
  - $\frac{d^2x(t)}{dt^2} = -\frac{D_f}{m} V \frac{dx(t)}{dt} = -\frac{D_f}{m} V v_x$
  - $\frac{d^2y(t)}{dt^2} = -\frac{D_f}{m} V \frac{dy(t)}{dt} - g = -\frac{D_f}{m} V v_y - g$
  - x-direction velocity: $v_x = \frac{dx(t)}{dt}$ $(m/s)$
  - y-direction velocity: $v_y = \frac{dy(t)}{dt}$ $(m/s)$
  - x-direction acceleration: $a_x = \frac{d^2x(t)}{dt^2}$ $(m/s^2)$
  - y-direction acceleration: $a_y = \frac{d^2y(t)}{dt^2}$ $(m/s^2)$
  - $V = \sqrt{v_x^2 + v_y^2}$ is the velocity of the sphere $(m/s)$
  - mass of the sphere: $m = \rho_s V_s$ $(kg)$

# Drag Force

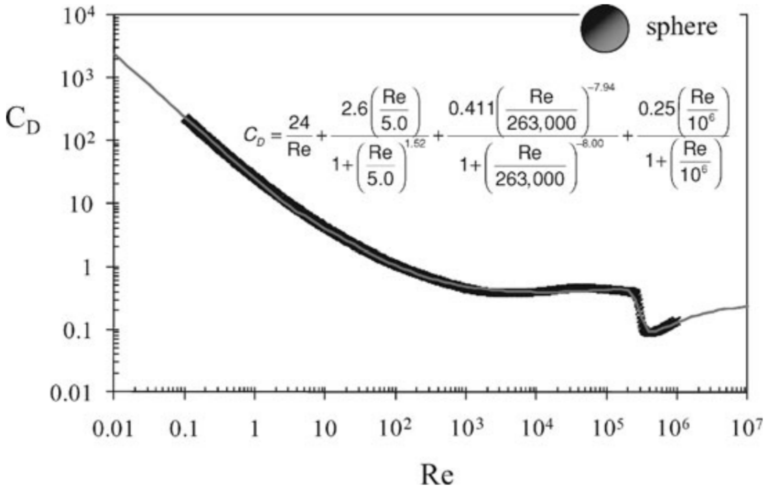
- The drag force parameter is: $D_f = \frac{\rho C_d A}{2}$ $(kg/m)$
  - $\rho$ is the fluid density $(kg/m^3)$
  - $C_d$ is the drag coefficient (dimensionless)
    - a function of Reynolds number: $Re = \frac{\rho V D}{\mu}$
    - $D$ $(m)$, is the diameter of the sphere
    - $\mu$ $(kg \cdot m/s)$, is the fluid viscosity
  - $A$ is the cross sectional area (frontal area)
    - For a sphere $A = \frac{\pi D^2}{4}$ $(m^2)$

# Drag Coefficient Correlation for a Sphere

- Drag coefficient correlation for a sphere:
  - $C_d = \frac{24}{\mathrm{Re}} + \frac{2.6\left(\frac{\mathrm{Re}}{5.0}\right)}{1+\left(\frac{\mathrm{Re}}{5.0}\right)^{1.52}} + \frac{0.411\left(\frac{\mathrm{Re}}{263{,}000}\right)^{-7.94}}{1+\left(\frac{\mathrm{Re}}{263{,}000}\right)^{-8.0}} + \frac{0.25\left(\frac{\mathrm{Re}}{10^6}\right)}{1+\left(\frac{\mathrm{Re}}{10^6}\right)}$
  - valid for $0.01 \leq \mathrm{Re} \leq 10^6$
- Equation (8.83) on p. 624 in [Mor13][1]

[1][Mor13] F. A. Morrison, *An Introduction to Fluid Mechanics*, Cambridge University Press, Cambridge, 2013

# Correlation Comparison with Data




[2] [Mor13] F. A. Morrison, *An Introduction to Fluid Mechanics*, Cambridge University Press, Cambridge, 2013

# Summary of Math Model Parameters

- Material Properties
  - Fluid density $\rho$ $(kg/m^3)$
  - Fluid viscosity $\mu$ $(kg \cdot m/s = Pa \cdot s = N \cdot s/m^2)$
  - Sphere density $\rho_s$ $(kg/m^3)$
- Geometry
  - Sphere diameter $D$ $(m)$, volume is $V_s = \frac{4\pi r^3}{3} = \frac{\pi D^3}{6}$ $(m^3)$
- Initial/Boundary Conditions
  - Muzzle velocity $V$ $(m/s)$
  - Angle $\theta$ $(^\circ)$, $v_x = \cos(\theta)V$ and $v_y = \sin(\theta)V$
- Correlation
  - Drag coefficient $C_d(Re)$, a function of Reynolds number
- Constants
  - Gravity $g$ $(m^2/s)$