The Cannon Problem

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ME498/599 Ballistic Problem with Air Resistance

- Sensitivity analysis
- Uncertainty quantification
- Uncertainty propagation

Math Model

- The equations of motion (Newton's Second Law $ma = \sum F$) are:
  - $\frac{d^2x(t)}{dt^2} = -\frac{D}{m}\frac{dx(t)}{dt}$
  - $\frac{d^2y(t)}{dt^2} = -\frac{D}{m}\frac{dy(t)}{dt} - g$
  - x-direction velocity: $v_x = \frac{dx(t)}{dt}$
  - y-direction velocity: $v_y = \frac{dy(t)}{dt}$
  - x-direction acceleration: $a_x = \frac{d^2x(t)}{dt^2}$
  - y-direction acceleration: $a_y = \frac{d^2y(t)}{dt^2}$

Drag Force

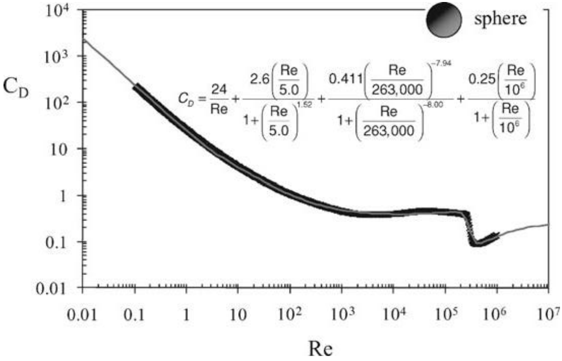
- The drag force is: $D = \frac{C_d A \rho V^2}{2}$
  - C_d is the drag coefficient, a function of $Re = \frac{\rho V D}{\mu}$
    - D is the diameter of the sphere and μ is the fluid viscosity
  - A is the cross sectional area (frontal area)
    - For a sphere $A = \frac{\pi D^2}{4}$
  - ρ is the fluid density
  - $V = \sqrt{v_x^2 + v_y^2}$ is the velocity

Correlation for Drag Coefficient for a Sphere

- Drag coefficient correlation for a sphere:
  - $C_d = \frac{24}{\mathrm{Re}} + \frac{2.6\left(\frac{\mathrm{Re}}{5.0}\right)}{1+\left(\frac{\mathrm{Re}}{5.0}\right)^{1.52}} + \frac{0.411\left(\frac{\mathrm{Re}}{263,000}\right)^{-7.94}}{1+\left(\frac{\mathrm{Re}}{263,000}\right)^{-8.0}} + \frac{0.25\left(\frac{\mathrm{Re}}{10^6}\right)}{1+\left(\frac{\mathrm{Re}}{10^6}\right)}$
- Equation (8.83) on p. 624 in [Mor13][1]

[1] [Mor13] F. A. Morrison, *An Introduction to Fluid Mechanics*, Cambridge University Press, Cambridge, 2013

Correlation Comparison with Data



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[2] [Mor13] F. A. Morrison, *An Introduction to Fluid Mechanics*, Cambridge University Press, Cambridge, 2013

Summary of Math Model Parameters

- Material Properties
  - Fluid density ρ
  - Fluid viscosity μ
  - Sphere density ρ_s
- Geometry
  - Sphere diameter D, volume is $Vol = \frac{4\pi r^3}{3} = \frac{\pi D^3}{6}$
- Boundary Conditions
  - Muzzle velocity V
  - Angle θ
- Correlation
  - Drag coefficient $C_d(Re)$, a function of Reynolds number
- Constants
  - Gravity g