Variance Based Decomposition/Sobol' Indices

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Variance Based Decomposition

- Identify the fraction of the variance in the output that can be attributed to an individual input variable alone or with interaction effects
  - Sobol' indices

Main Effects

- Output Y attributed to input X_i alone:

$$S_i = \frac{\text{var}(E(Y|X_i))}{\text{var}(Y)}$$

- Varies between 0 and 1 and:

$$\sum S_i = 1$$

Total Effects

- Fraction of the uncertainty in output Y attributed to X_i and its interactions with the other variables:

$$T_i = \frac{E[\text{var}(Y|X_{-i})]}{\text{var}(Y)}$$

- $T_i \geq 0$ and is not bounded by 1

Directly Calculate Sobol Indices

- Use variance based decomposition method in Dakota
  - See slide 22 in Dakota Training: Sensitivity Analysis
  - Expensive: number of `samples` X (number of `variables` + 2)

```
method
  sampling
    sample_type = lhs
    samples     = <int> # 10 X number of variables
    seed        = <int> # 5-6 digit number
    variance_based_decomp
```

Calculate Using PCE Surrogate Model

- Construct PCE using LHS data for VBD
  - See slide 26 in Uncertainty Quantification
  - See 1:32 in the training video: Uncertainty Quantification

```
method
  polynomial_chaos
    expansion_order_sequence    = <int>
    collocation_points_sequence = <int>
    orthogonal_matching_pursuit
    cross_validation
    import_build_points_file    = <str>  # use tabular data
    variance_based_decomp
```